

Complex Ginzburg-Landau Equation and the Tachyonic Mass Problem

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Abstract

The Standard Model of particle physics postulates that the $(\text{mass})^2$ term of the Higgs potential is negative. This choice is considered unnatural and leads to the *tachyonic mass problem*. It is known that the formulation of the Higgs mechanism relies on the standard Ginzburg-Landau equation describing equilibrium phase transitions. It is also known that the Complex Ginzburg-Landau equation (CGLE) is a universal model of complex dynamics outside equilibrium. This brief note suggests that the tachyonic mass problem goes away upon switching from the standard Ginzburg-Landau equation to the CGLE.

Key words: Higgs mechanism, tachyonic mass problem, complex Ginzburg-Landau equation, nonequilibrium dynamics.

The standard Ginzburg-Landau potential underlying the Higgs mechanism of spontaneous symmetry breaking is given by,

$$V_{GLE}(\varphi) = \mu^2 \varphi^\dagger \varphi + \lambda (\varphi^\dagger \varphi)^2 \quad (1)$$

where a positive self-interaction coupling ($\lambda > 0$) forces (1) to be bounded below as the field goes to infinity, $(\varphi^\dagger \varphi)^{1/2} \rightarrow \infty$. The potential (1) has a minimum at

$$(\varphi^\dagger \varphi)_{GLE} = \frac{-\mu^2}{2\lambda} = v^2 \quad (2)$$

in which the mass parameter $\mu^2 < 0$ and v represents the vacuum expectation value of the Higgs boson. It is known that the CGLE defines the generic dynamics of a complex order parameter z and assumes the form [1]

$$\partial_t z = az + (1 + ic_1)\Delta z - (1 - ic_3)z|z|^{2\sigma} \quad (3)$$

where a and σ are positive and c_1, c_3 are real. In the absence of spatial dependence ($\nabla z = \Delta z = 0$) and upon taking $c_3 = 0, \sigma = 1, z = \text{real}$, (3) can be rescaled to [1-3]

$$\partial_t z = -\frac{\partial V(z)}{\partial z} = \alpha z - \beta z^3 \quad (4)$$

in which $\alpha > 0, \beta > 0$. Side by side evaluation of (1) and (4) yields

$$V(z) = -\frac{\alpha}{2}z^2 + \frac{\beta}{4}z^4 \quad (5)$$

$$z = (\varphi^\dagger \varphi)^{1/2} \quad (6)$$

$$-\frac{\alpha}{2} = -\mu^2 \quad (7)$$

$$\frac{\beta}{4} = \lambda \quad (8)$$

By (5)-(8), the CGLE potential of the Higgs boson gets changed from (1) to,

$$V_{\text{CGLE}}(\varphi) = -\mu^2 \varphi^\dagger \varphi + \lambda (\varphi^\dagger \varphi)^2 \quad (9)$$

with a positive mass square $\mu^2 = \alpha/2 > 0$ and a minimum at

$$(\varphi^\dagger \varphi)_{\text{CGLE}} = \frac{\mu^2}{2\lambda} = v^2 \quad (10)$$

References

1. <https://arxiv.org/pdf/cond-mat/0106115.pdf>

2. Equations (4.1a) to (4.1d) in:

<https://courses.physics.ucsd.edu/2019/Fall/physics239/GOODIES/HH77.pdf>

3. Paragraph 2.7 in Strogatz S. H., *Nonlinear Dynamics and Chaos*, Westview Press, 1994.